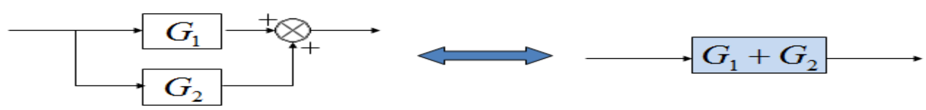
Block Diagram

There are occasions when there is interaction between the control loops and, for the purpose of analysis, it becomes necessary to re-arrange the block diagram configuration. This can be undertaken using Block Diagram Transformation Theorems.

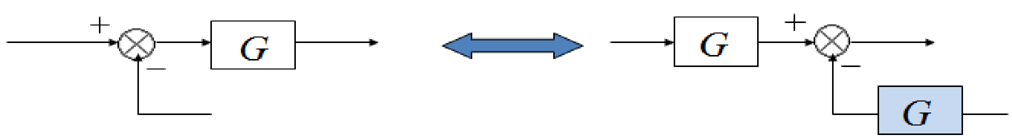
Block Diagram Transformation Theorems include many rules and these rules are:

1. Combining blocks in cascade

2. Combining blocks in parallel

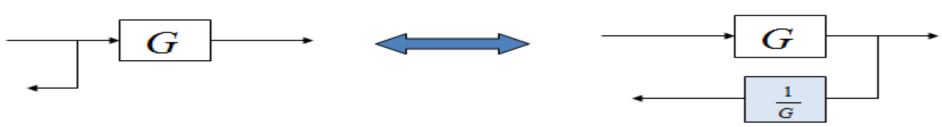


3. Moving a summing point behind a block



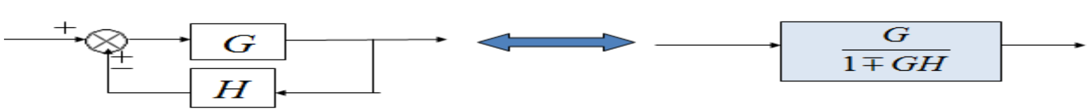
3. Moving a summing point ahead of a block

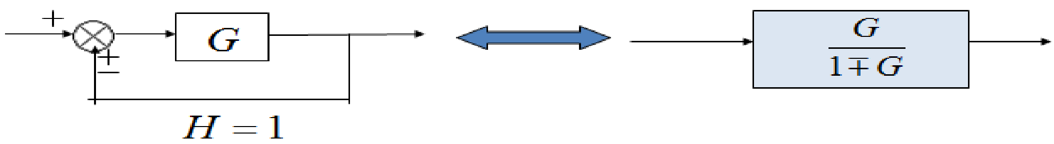
4. Moving a pickoff point behind a block



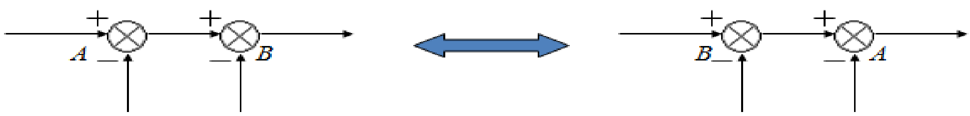
5. Moving a pickoff point ahead of a block

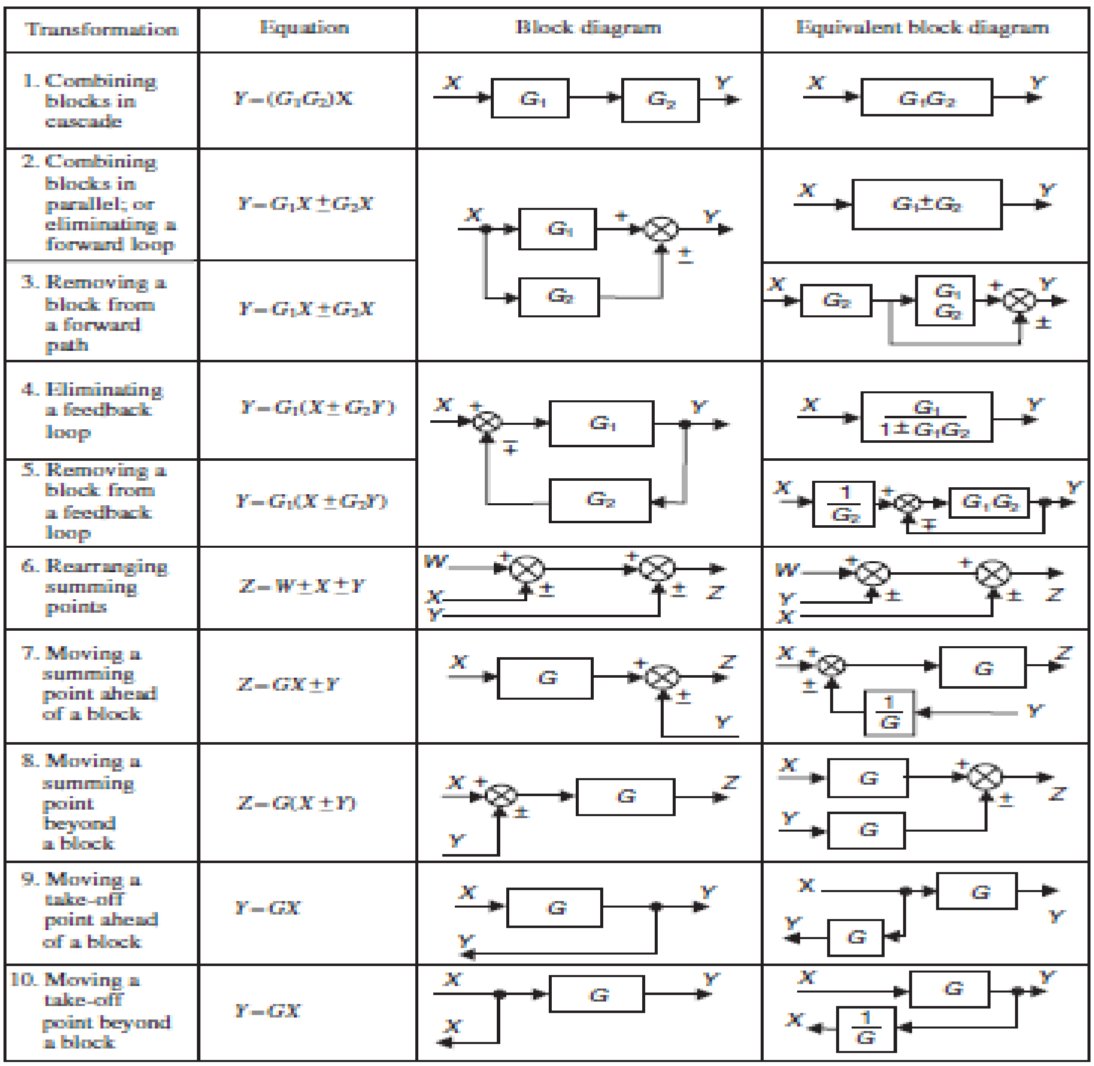
6. Eliminating a feedback loop





7. Swap with two neighboring summing points

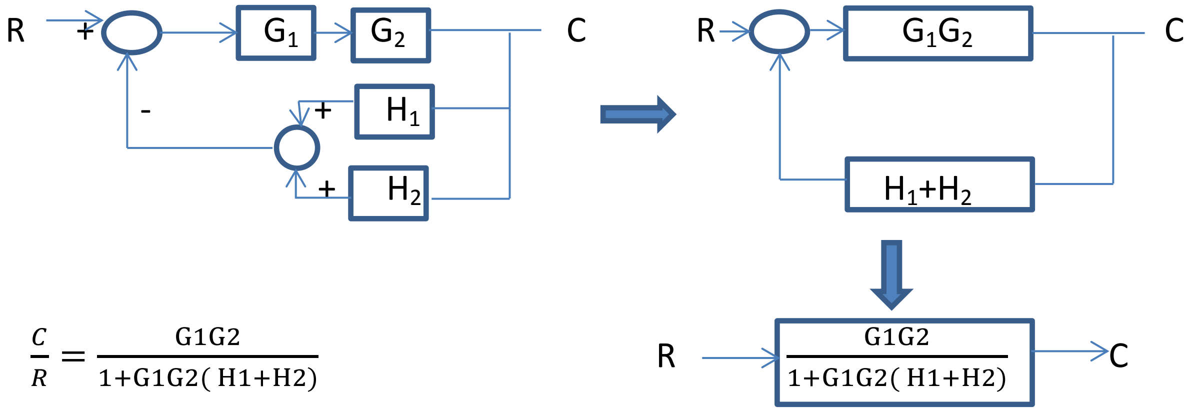




Transformation	Equation	Block diagram	Equivalent block diagram
1. Combining blocks in cascade	$Y=(G_1G_2)X$		
2. Combining blocks in parallel; or eliminating a forward loop	$Y=G_1X\pm G_2X$		
3. Removing a block from a forward path	$Y=G_1X\pm G_2X$		
4. Eliminating a feedback loop	$Y=G_1(X\pm G_2Y)$		
5. Removing a block from a feedback loop	$Y=G_1(X\pm G_2Y)$		
6. Rearranging summing points	$Z=W\pm X\pm Y$		
7. Moving a summing point ahead of a block	$Z=GX\pm Y$		
8. Moving a summing point beyond a block	$Z=G(X\pm Y)$		
9. Moving a take-off point ahead of a block	$Y=GX$		
10. Moving a take-off point beyond a block	$Y=GX$		

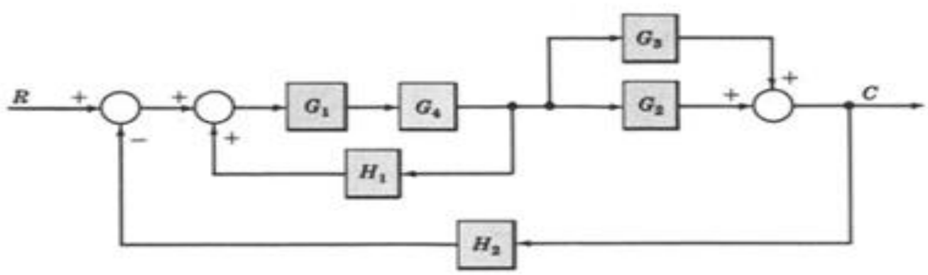
Table (3.1) Block Diagram transformation theorems

Ex1: Find the transfer function of the following block diagrams



$$\frac{C}{R} = \frac{G1G2}{1+G1G2(\,H1+H2)}$$

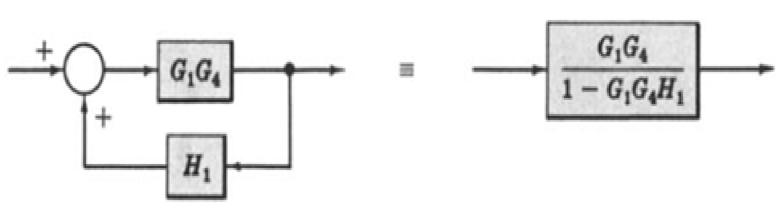
Ex2: Find the transfer function of the following block diagrams



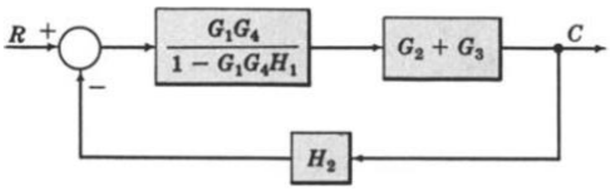
Step 1: Combine all cascade blocks using Transformation 1.

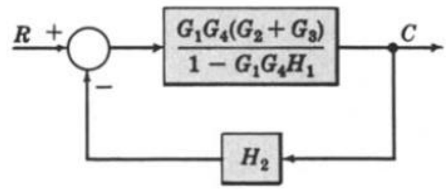
Step 2: Combine all parallel blocks using Transformation 2.

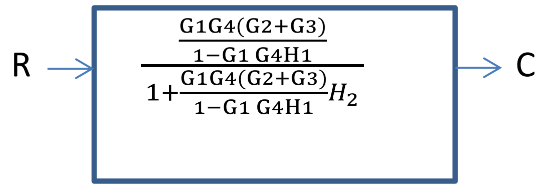
Step 3: Eliminate all minor feedback loops



Step 4: Shift summing points to the left and takeoff points to the right of the major loop.



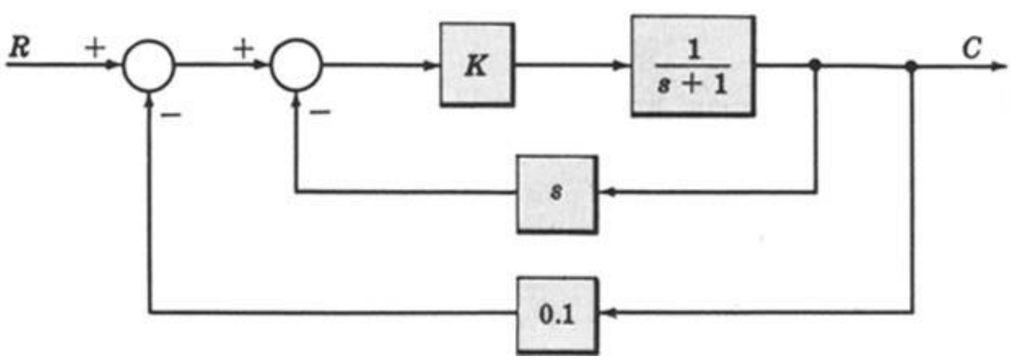




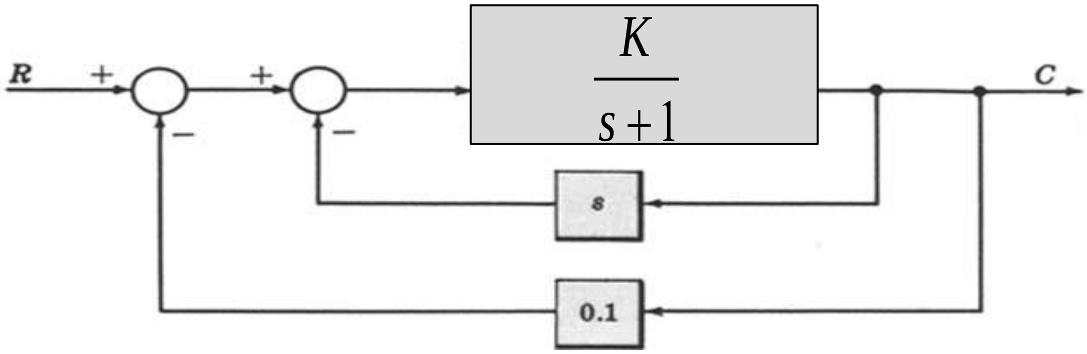
$$\therefore \frac{C}{R} = \frac{\frac{G1G4(G2+G3)}{1-G1\,G4H1}}{1+\frac{G1G4(G2+G3)}{1-G1\,G4H1}H_2}$$

Ex3: For the system represented by the following block diagram determine:

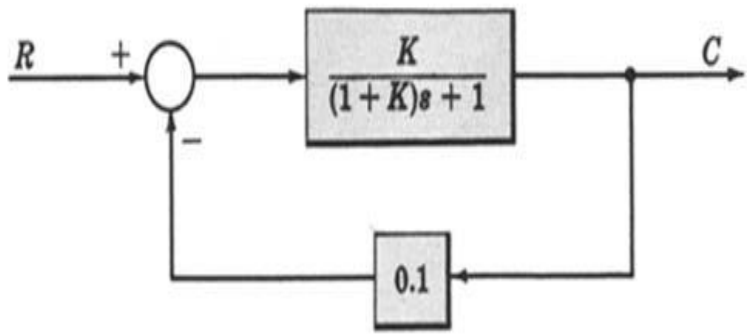
1. Open loop transfer function
2. Feed Forward Transfer function
3. control ratio
4. feedback ratio
5. error ratio
6. closed loop transfer function
7. characteristic equation



Sol: First we will reduce the given block diagram



$$\frac{G}{1+GH}=\frac{\frac{K}{s+1}}{1+\frac{K}{s+1}s}$$



1. Open loop transfer function $\frac{B(s)}{E(s)}=G(s)H(s)$ $=\frac{0.1K}{(1+K)s+1}$

2. Feed Forward Transfer function $\frac{C(s)}{E(s)}=G(s)$ $=\frac{K}{(1+K)s+1}$

3.control ratio $\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}$ $=\frac{\frac{K}{(1+K)s+1}}{1+0.1\frac{K}{(1+K)s+1}}$

4. feedback ratio $\frac{B(s)}{R(s)}=\frac{G(s)H(s)}{1+G(s)H(s)}$ $=\frac{\frac{0.1K}{(1+K)s+1}}{1+0.1\frac{K}{(1+K)s+1}}$

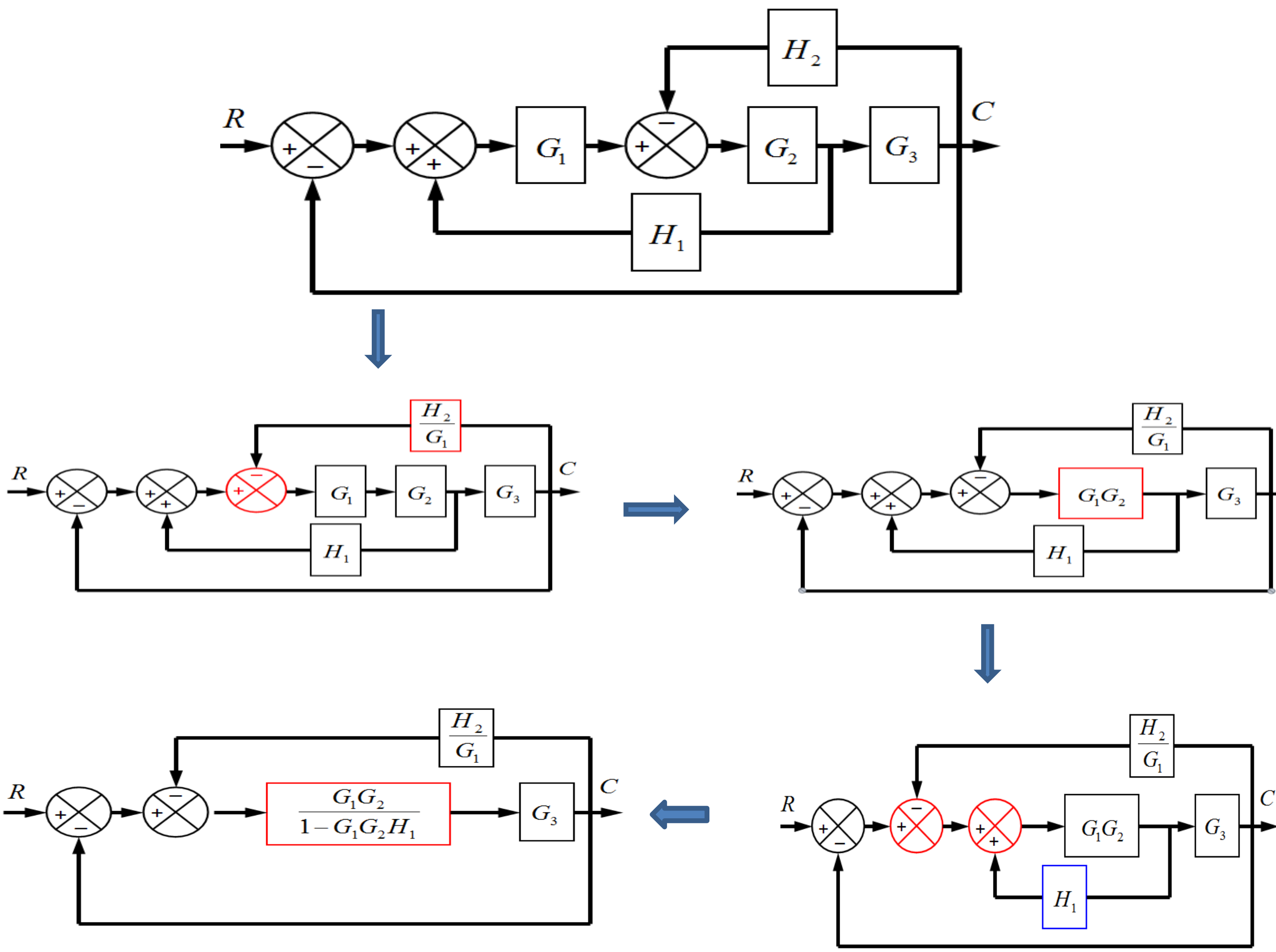
5. error ratio $\frac{E(s)}{R(s)}=\frac{1}{1+G(s)H(s)}$ $=\frac{1}{1+0.1\frac{K}{(1+K)s+1}}$

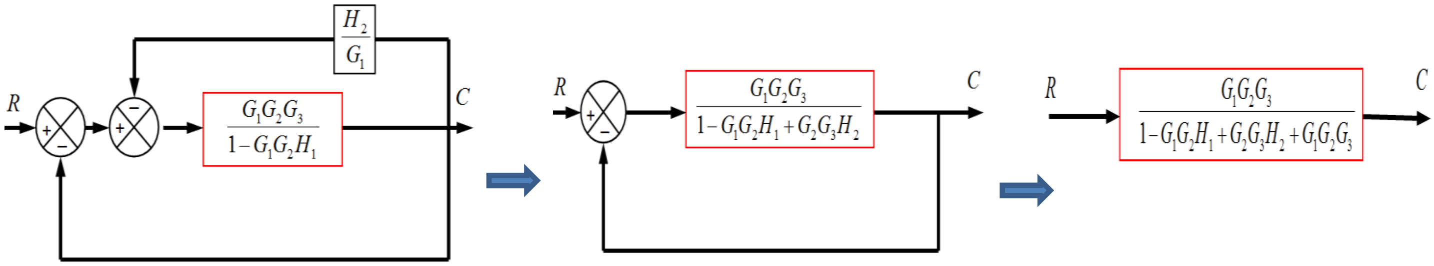
6. closed loop transfer function $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$ $= \frac{\frac{K}{(1+K)s+1}}{1+0.1\frac{K}{(1+K)s+1}}$

7. characteristic equation $1+G(s)H(s)=0$ $1+0.1\frac{K}{(1+K)s+1} = 0$

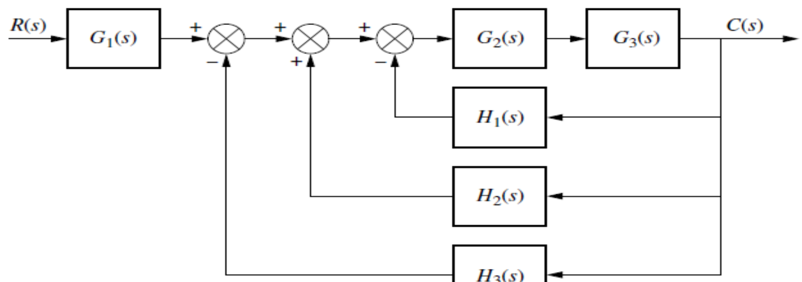
$$= (1+K)s + 0.1K = 0$$

Ex4: Find the transfer function of the following block diagrams

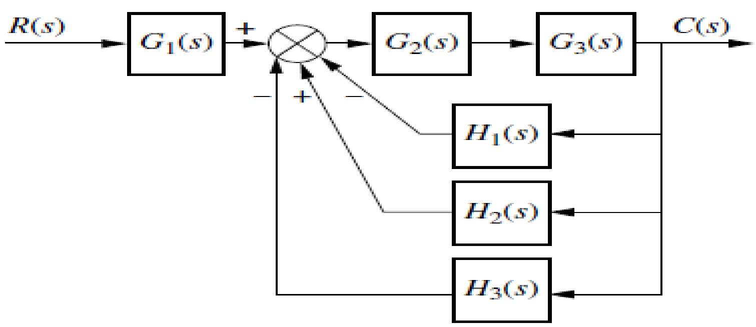




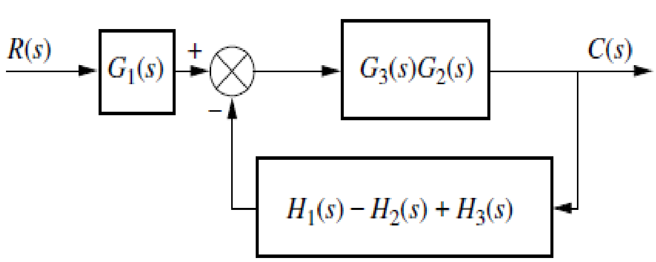
Ex5: Reduce the Block Diagram



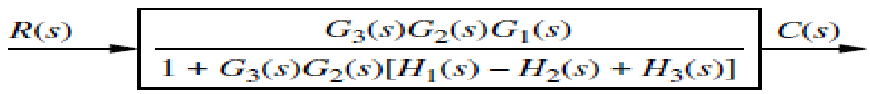
First, the three summing junctions can be collapsed into a single summing junction,



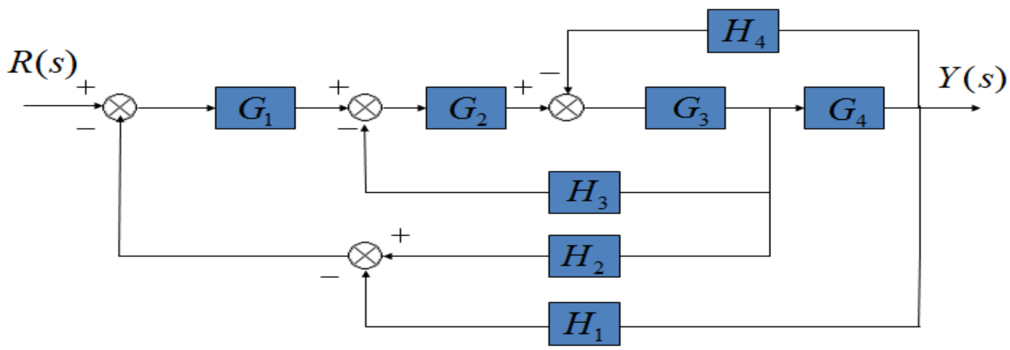
Second, recognize that the three feedback functions, $H_1(s)$, $H_2(s)$, and $H_3(s)$, are connected in parallel. They are fed from a common signal source, and their outputs are summed. Also recognize that $G_2(s)$ and $G_3(s)$ are connected in cascade.



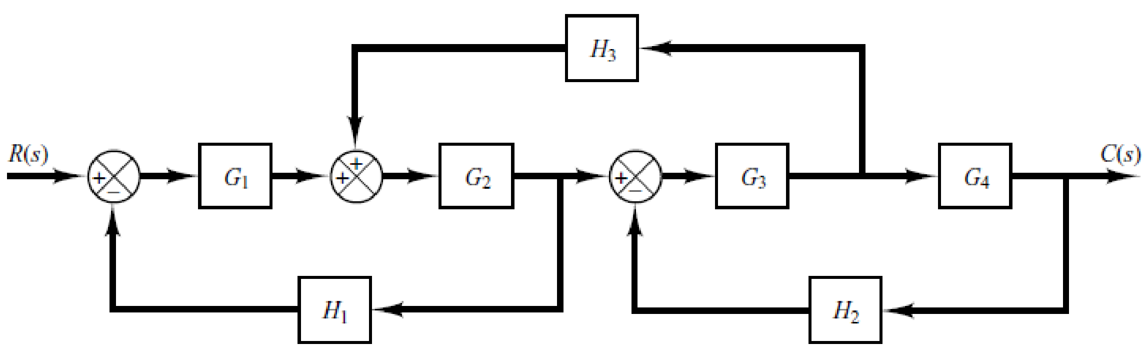
Finally, the feedback system is reduced and multiplied by $G_1(s)$ to yield the equivalent transfer function shown in Figure



H.W1: Find the transfer function of the following block diagrams

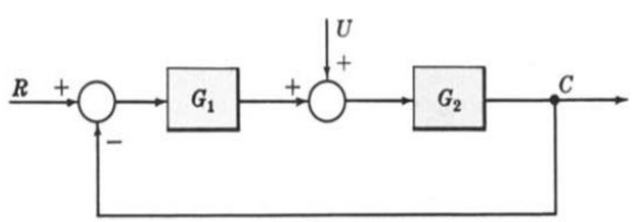


H.W2: Simplify the block diagram then obtain the close-loop transfer function C(S)/R(S).



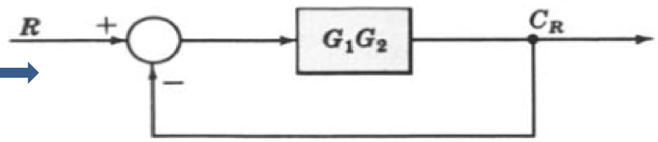
Multiple Input System

EX1: Determine the output C due to inputs R and U using the Superposition Method



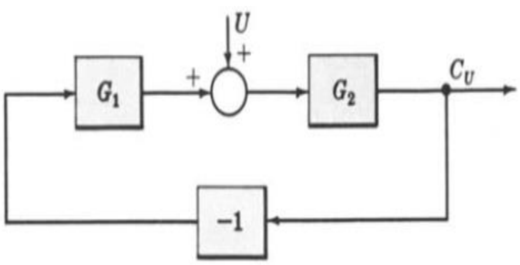
sol: according to the principles of superposition, then, we must try to find the o/p considering one i/p at time

Step 1: Put $U = 0$.
Step 2: The system reduces to

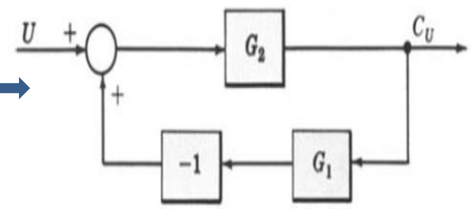


Step 3: the output C_R due to input R is $C_R = [G_1G_2/(1 + G_1G_2)]R$.

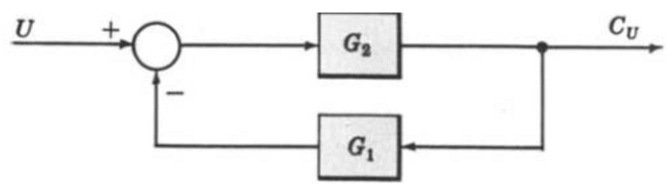
Step 4a: Put $R = 0$.
Step 4b: Put -1 into a block, representing the negative feedback effect:



Rearrange the block diagram:



Let the -1 block be absorbed into the summing point:



Step 4c: the output C_U due to input U is $C_U = [G_2/(1 + G_1G_2)]U$.

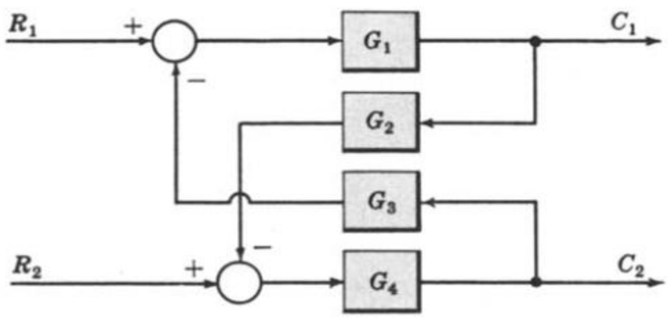
Step 5: The total output is $C = C_R + C_U$

$$= \left[\frac{G_1G_2}{1 + G_1G_2}\right]R + \left[\frac{G_2}{1 + G_1G_2}\right]U$$

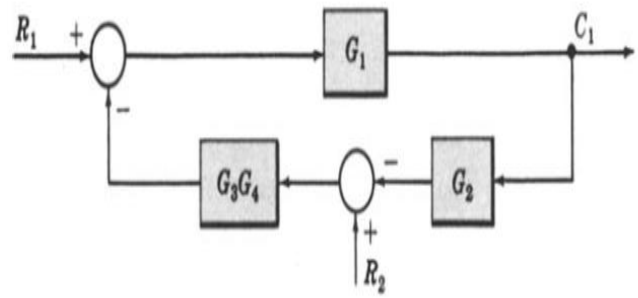
$$= \left[\frac{G_2}{1 + G_1G_2}\right][G_1R + U]$$

Multi-Input Multi-Output System

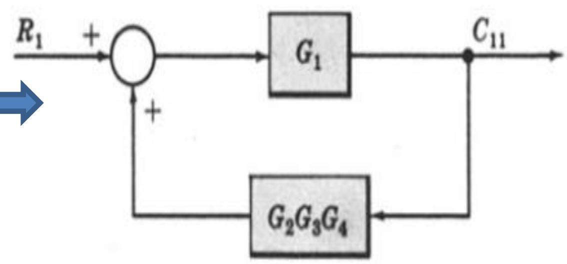
Ex1: Determine C1 and C2 due to R1 and R2.



First ignoring the output C_2.

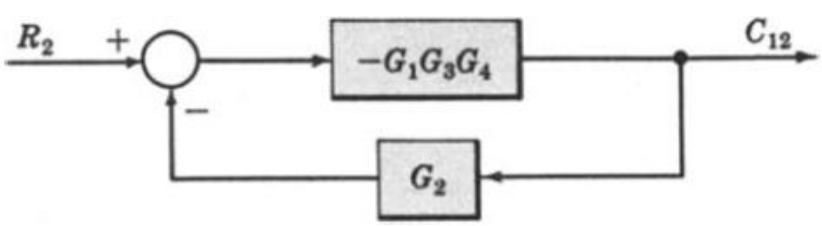


Letting $R_2 = 0$ and combining the summing points,



Hence C_{11}, the output at C_1 due to R_1 alone, is $C_{11} = G_1 R_1/(1 - G_1 G_2 G_3 G_4)$.

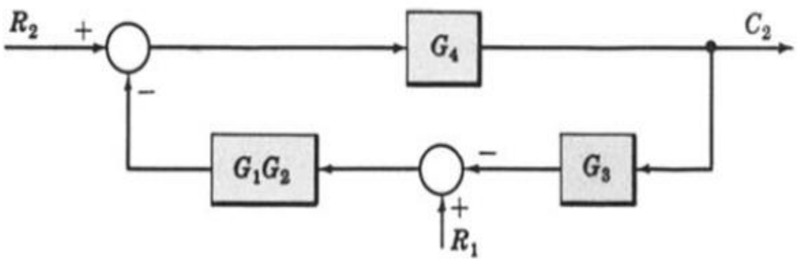
For $R_1 = 0$,



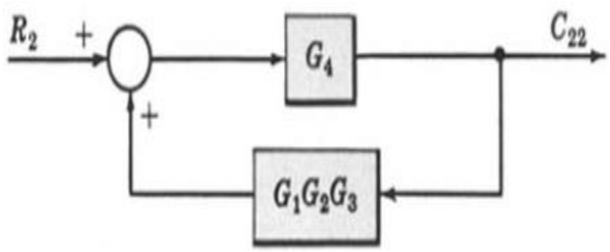
Hence $C_{12} = -G_1 G_3 G_4 R_2/(1 - G_1 G_2 G_3 G_4)$ is the output at C_1 due to R_2 alone.

Thus $C_1 = C_{11} + C_{12} = (G_1 R_1 - G_1 G_3 G_4 R_2)/(1 - G_1 G_2 G_3 G_4)$

Now we reduce the original block diagram, ignoring output C_1.

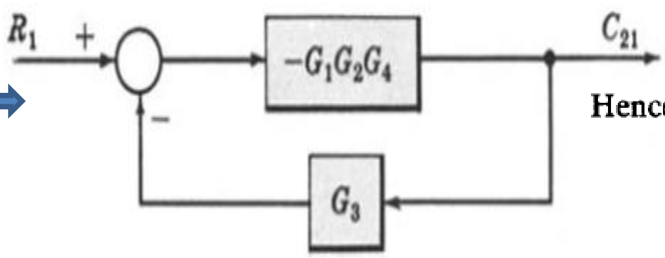


When R1 = 0 ⇒



Hence $C_{22} = G_4 R_2/(1 - G_1G_2G_3G_4)$

When R2 = 0 ⇒



Hence $C_{21} = -G_1G_2G_4R_1/(1 - G_1G_2G_3G_4)$

Finally, $C_2 = C_{22} + C_{21} = (G_4R_2 - G_1G_2G_4R_1)/(1 - G_1G_2G_3G_4)$

Signal Flow Graphs

The block diagram is useful for graphically representing control system dynamics and is used extensively in the analysis and design of control systems. An alternate approach for graphically representing control system dynamics is the signal flow graph approach, due to S. J. Mason. It is noted that the signal flow graph approach and the block diagram approach yield the same information and one is in no sense superior to the other.

Definitions: Before we discuss signal flow graphs, we must define certain terms:

- **Node:** A node is a point representing a variable or signal.
- **Transmittance:** The transmittance is a real gain or complex gain between two nodes. Such gains can be expressed in terms of the transfer function between two nodes.
- **Branch:** A branch is a directed line segment joining two nodes. The gain of a branch is a transmittance.
- **Input node or source:** An input node or source is a node that has only outgoing branches.
- **Output node or sink:** An output node or sink is a node that has only incoming branches.
- **Mixed node:** A mixed node is a node that has both incoming and outgoing branches.
- **Path:** A path is a traversal of connected branches in the direction of the branch arrows.
- **Loop:** A loop is a closed path.
- **Loop gain:** The loop gain is the product of the branch transmittances of a loop.
- **Non touching loops:** Loops are nontouching if they do not possess any common nodes.
- **Forward path:** A forward path is a path from an input node (source) to an output node (sink) that does not cross any nodes more than once.
- **Forward path gain:** *A* forward path gain is the product of the branch transmittances of a forward path.

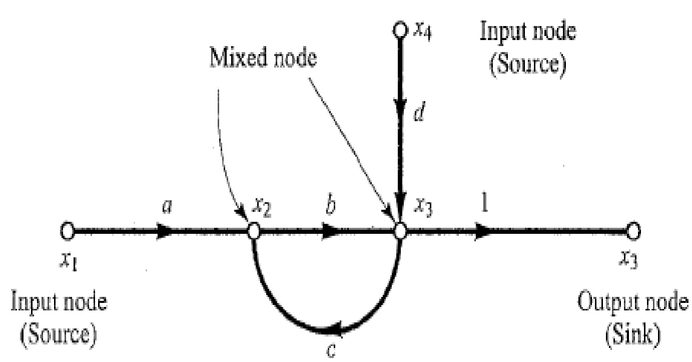


Figure of Signal flow graph

Signal Flow Graph Algebra

A signal flow graph of a linear system can be drawn using the foregoing definitions. In doing so, we usually bring the input nodes (sources) to the left and the output nodes (sinks) to the right. The independent and dependent variables of the equations become the input nodes (sources) and output nodes (sinks), respectively. The branch transmittances can be obtained from the coefficients of the equations. To determine the input-output relationship, we may use Mason's formula, which will be given later, or we may reduce the signal flow graph to a graph containing only input and output nodes. To accomplish this, we use the following rules:

1. The value of a node with one incoming branch, as shown in Figure 3-1(a), is $x2 = \boldsymbol{ax1}$.

2. The total transmittance of cascaded branches is equal to the product of all the branch transmittances. Cascaded branches can thus be combined into a single branch by multiplying the transmittances, as shown in Figure 3-1(b).

3. Parallel branches may be combined by adding the transmittances, as shown in Figure 3-1(c).

4. A mixed node may be eliminated, as shown in Figure 3-1(d).

5. A loop may be eliminated, as shown in Figure 3-1(e). Note that $x3 = bx2$, $x2 = \boldsymbol{ax1} + cx3$

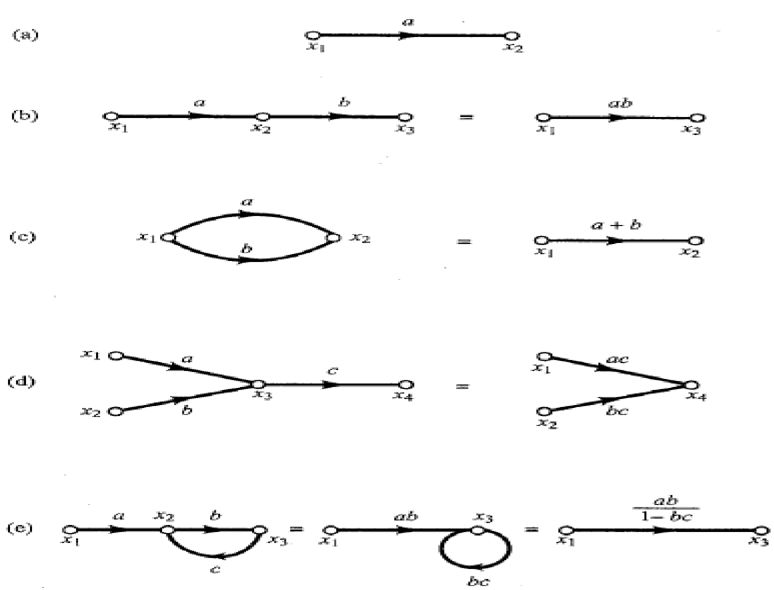


Figure 3-1 Signal flow graphs and simplifications

Hence $x_3 = abx_1 + bcx_3$ (1)

$$x_3 = \frac{ab}{1-bc}x_1 \qquad (2)$$

Equation (1) corresponds to a diagram having a self-loop of transmittance *bc*. Elimination of the self-loop yields Equation (2), which clearly shows that the overall transmittance is *ab/(l - bc)*.

Signal Flow Graphs of Control Systems:

Some signal flow graphs of simple control systems are shown in Figure 3-2. For such simple graphs, the closed-loop transfer function *C(s)/R(s)*[or *c(s)/N(s)]*can be obtained easily by inspection. For more complicated signal flow graphs, Mason's gain formula is quite useful.

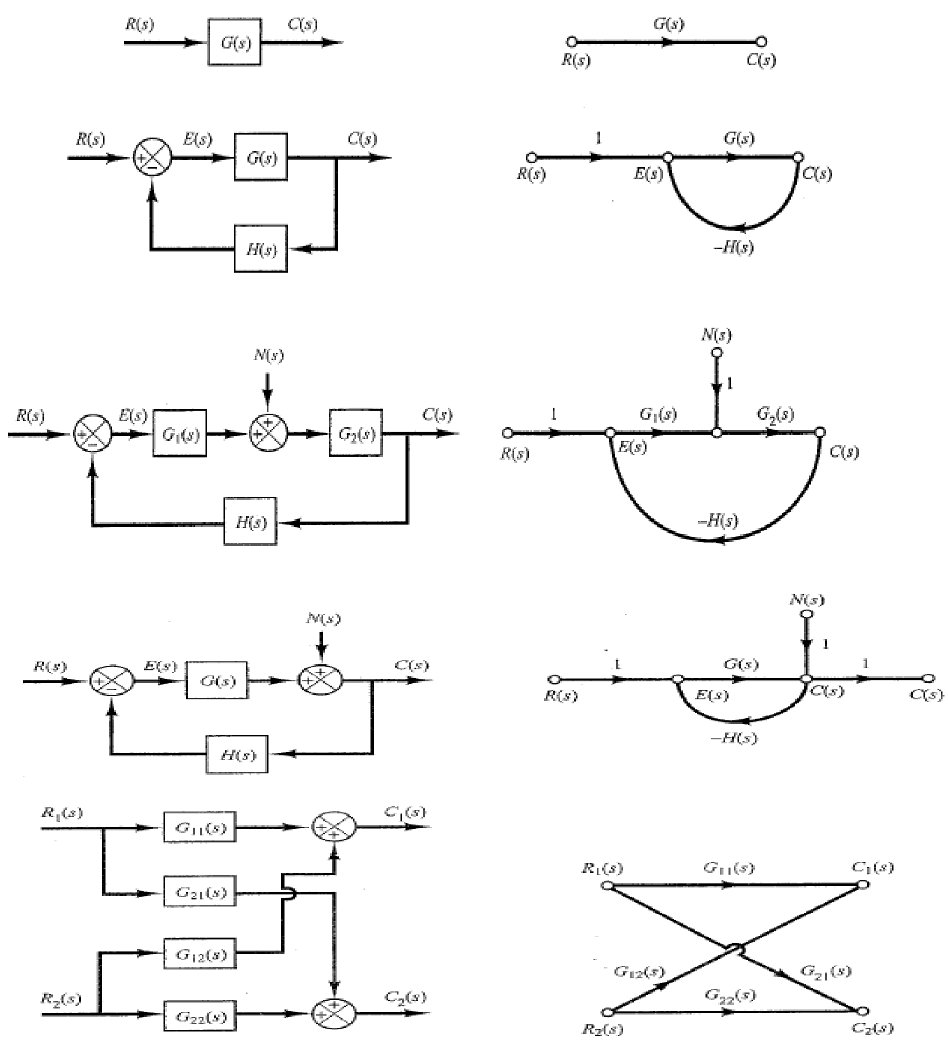


Figure 3-2 Block diagrams and corresponding signal flow graphs

Mason's Gain Formula for Signal Flow Graphs

The block diagram reduction technique requires successive application of fundamental relationships in order to arrive at the system transfer function. On the other hand, Mason's rule for reducing a signal-flow graph to a single transfer function requires the application of one formula. The formula was derived by S. J. Mason when he related the signal-flow graph to the simultaneous equations that can be written from the graph.

The transfer function, ***C(s)/R(s),*** of a system represented by a signal-flow graph is:

$$\boxed{\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}}$$

Where

n = number of forward paths.

P_i = the i^{th} forward-path gain.

Δ = Determinant of the system or is a series of summation of the untouched loops

Δ_i = Determinant of the i^{th} forward path

Δ is called the signal flow graph determinant or characteristic function

Δ = 1- (sum of all individual loop gains) + (sum of the products of the gains of all possible two loops that do not touch each other) – (sum of the products of the gains of all possible three loops that do not touch each other) + … and so forth with sums of higher number of non-touching loop gains

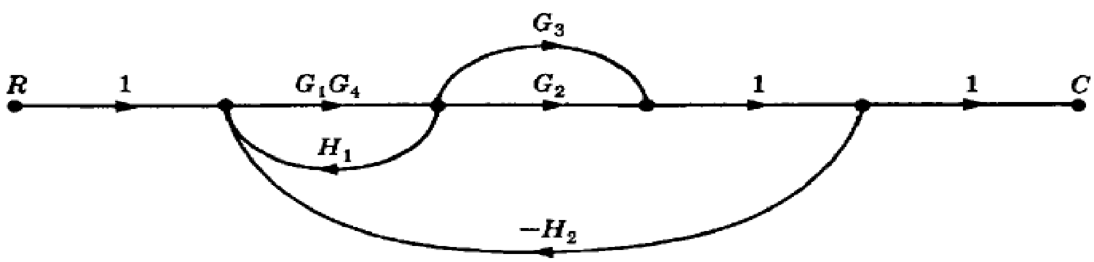
Δ_i = value of Δ for the part of the block diagram that does not touch the i-th forward path (Δ_i = 1 if there are no non-touching loops to the i-th path.)

Solution steps by Mason's rule

1. Calculate forward path gain P_i for each forward path i.
2. Calculate all loop transfer functions
3. Consider non-touching loops 2 at a time
4. Consider non-touching loops 3 at a time
5. Etc

6. Calculate Δ from steps 2,3,4 and 5
7. Calculate Δ_i as portion of Δ not touching forward path *i*

Example1: Apply Mason's Rule to calculate the transfer function of the system represented by following Signal Flow Graph



There are two forward paths:

$P_1 = G_1G_2G_4$ $\qquad$ $P_2 = G_1G_3G_4$

Therefore

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

There are three feedback loops

$$L_1 = G_1G_4H_1, \quad L_2 = -G_1G_2G_4H_2, \quad L_3 = -G_1G_3G_4H_2$$

There are no non-touching loops, therefore

Δ = 1- (sum of all individual loop gains)

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

$$\Delta = 1 - (G_1G_4H_1 - G_1G_2G_4H_2 - G_1G_3G_4H_2)$$

Eliminate forward path-1

Δ_1 = 1- (sum of all individual loop gains)+... $\longrightarrow$ $\Delta_1 = 1$

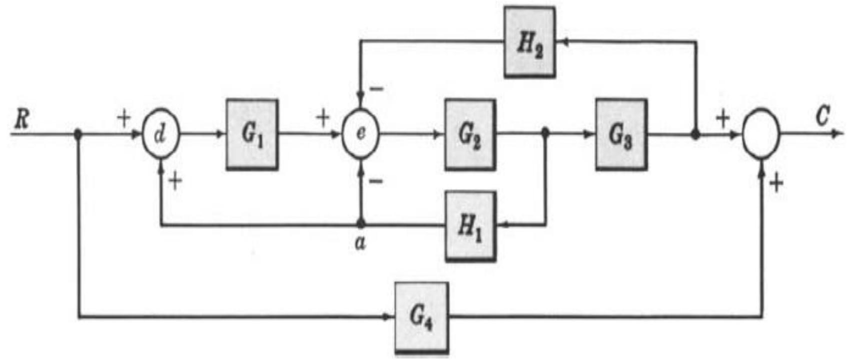
Eliminate forward path-2

Δ_2 = 1- (sum of all individual loop gains)+... $\longrightarrow$ $\Delta_2 = 1$

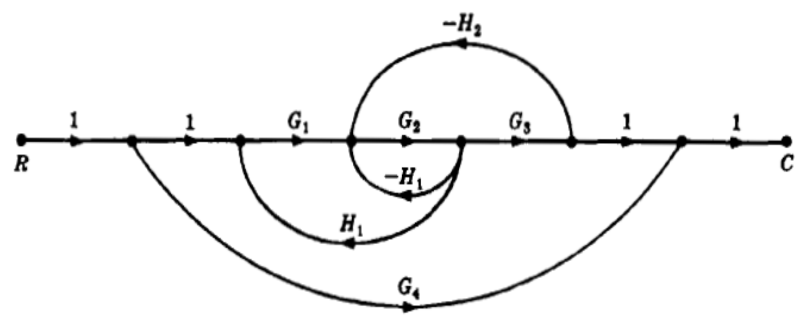
$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2} = \frac{G_1G_4(G_2 + G_3)}{1 - G_1G_4H_1 + G_1G_2G_4H_2 + G_1G_3G_4H_2}$$

Example-2: Determine the transfer function C/R for the block diagram below by signal flow graph techniques.

Sol:



- The signal flow graph of the above block diagram is shown below.



- There are two forward paths. The path gains are

$P_1 = G_1G_2G_3$ and $P_2 = G_4$

- The three feedback loop gains are

$\mathbf{L_1 = -G_2H_1,\ L_2 = G_1G_2H_1,\ L_3 = -G_2G_3H_2}$

- No loops are non-touching, hence

$$\Delta = 1 - (L_1 + L_2 + L_3)$$

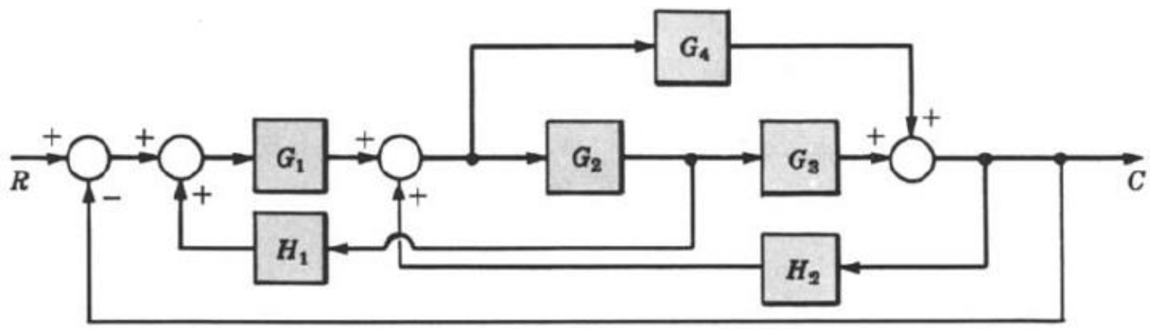
Because the loops touch the nodes of P1, hence $\Longrightarrow$ $\Delta_1 = 1$

Since no loops touch the nodes of P2, therefore $\Longrightarrow$ $\Delta_2 = \Delta$.

Hence the control ratio T = C/R is:

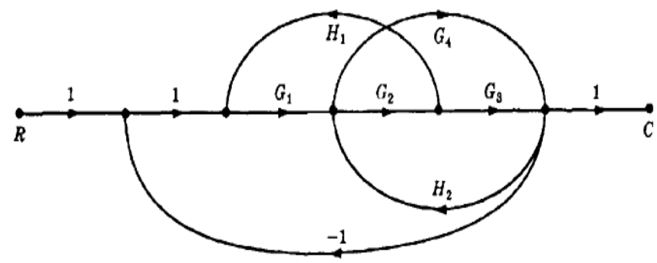
$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_4 + G_2G_4H_1 - G_1G_2G_4H_1 + G_2G_3G_4H_2}{1 + G_2H_1 - G_1G_2H_1 + G_2G_3H_2}$$

Example-3: Find the control ratio C/R for the system given below.



Sol:

- The signal flow graph is shown in the figure.



- The two forward path gains are

$$P_1 = G_1G_2G_3 \ \text{ and } \ P_2 = G_1G_4$$

- The five feedback loop gains are

  $L_1 = G_1G_2H_1$, $L_2 = G_2G_3H_2$, $L_3 = -G_1G_2G_3$, $L_4 = G_4H_2$, $L_5 = -G_1G_4$

There are no non-touching loops, hence

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$ $= 1 - G_1G_2H_1 - G_2G_3H_2 + G_1G_2G_3 - G_4H_2 + G_1G_4$

All feedback loops touches the two forward paths, hence ➡ $\Delta_1 = \Delta_2 = 1$

Hence the control ratio :

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_1G_2G_3 - G_1G_2H_1 - G_2G_3H_2 - G_4H_2 + G_1G_4}$$

Example-4: Find the control ratio C/R for the system given below